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$$x_1 = OP + PA = a + a \cos \alpha = a(1 + \cos \alpha); \text{ i. e., } OD = \frac{1}{2}x_1 = \frac{a}{2}(1 + \cos \alpha),$$

which gives as the polar equation of the locus referred to O as origin and OP as initial line,

$$\rho = \frac{a}{2}(1 + \cos \alpha),$$

a cardioid, with cusp at O passing through P and having OP as line of symmetry.

Also solved by J. Scheffer, A. H. Holmes, V. M. Spunar, G. B. M. Zerr, and J. W. Clawson.

CALCULUS.

264. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

The join of the center of curvature of a curve to the origin is at α to the initial line. Prove that with the usual notation:

$$\frac{d \alpha}{d \psi} \left[\left(\frac{dp}{d \psi} \right)^2 + \left(\frac{d^2 p}{d \psi^2} \right)^2 \right] = \frac{dp}{d \psi} \cdot \frac{d \rho}{d \psi}.$$

Solution by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

Denote the coordinates of the center of curvature by ξ, η . Then $\xi = x - \rho \sin \phi, \eta = y + \rho \cos \phi$, ρ being the radius of curvature and ϕ the angle formed by the tangent and positive x -axis.

$$\therefore \tan \alpha = \eta / \xi, \text{ and } d \alpha = \frac{\xi d \eta - \eta d \xi}{\xi^2 + \eta^2}.$$

Likewise, $\tan \phi = r \frac{d \theta}{d r}$, and $d \phi = \left[d \theta + r \left(\frac{d^2 \theta}{d r^2} \right) \right] \div \sec^2 \phi$, where ϕ = angle included by the tangent at (r, θ) , and the radius vector to the point (r, θ) .

Substituting the proper values of $\xi, \eta, d \xi, d \eta$ in $d \alpha$, expressed in polar coordinates, and we have the first product $d \alpha / d \phi$.

Also, on remembering that $\begin{cases} p = r \sin \phi \\ \varphi = \theta + \phi \end{cases}$, differentiate it twice, square every differential quotient as indicated by the proposition, add the like terms, reduce, and we have, readily,

$$\frac{d \alpha}{d \psi} \left[\left(\frac{dp}{d \psi} \right)^2 + \left(\frac{d^2 p}{d \psi^2} \right)^2 \right] = \frac{dp}{d \psi} \cdot \frac{d \rho}{d \psi}.$$

265. Proposed by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

Find two curves which possess the property that the tangents TP and TQ to the inner one always makes equal angles with the tangent TT' to the outer.

Remarks by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

In Salmon's *Conic Sections*, Sixth Edition, Articles 188-189, it is demonstrated that if through any point P of a conic section we draw tangents PT, QT to a confocal conic section, these tangents are equally inclined to the tangent at P . For every point P the outer conic through P , though confocal to the inner, has different axes.

If we wish to find a curve for all positions of P the problem assumes some difficulty.

For the ellipse, the equation to TP, TQ is

$$(a^2 - h^2)(y - k)^2 + 2(y - k)(x - h)hk + (b^2 - k^2)(x - h)^2 = 0,$$

where (h, k) are the coordinates of T .

$$2hk(y - k) + \{b^2 - a^2 + h^2 - k^2 \pm \sqrt{[(b^2 - a^2 + h^2 - k^2)^2 + 4h^2k^2]}\}(x - h) = 0,$$

gives the two lines making equal angles with TP, TQ .

The envelope of the line formed by using the plus sign subject to the condition obtained by using the minus sign gives the required curve.

If $a=b$, the ellipse becomes a circle. Then TT' is

$$ky + hx = h^2 + k^2 \dots (1).$$

$hy = kx$ is the perpendicular to (1).

The envelope of (1) subject to the condition $hy = kx$, is $x^2 + y^2 = 0$, or the center of the given circle.

266. Proposed by C. N. SCHMALL, New York City.

Show that the n th derivative of the fraction u/v can be expressed in the form of a determinant, u and v being functions of x .

Solution by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

As $u = \phi(x)$, $v = \psi(x)$, hence, $F(x) = \phi(x)/\psi(x) = u/v$, and therefore,

$$F'(x) = \frac{vu' - uv'}{v^2} = \frac{1}{v^2} \begin{vmatrix} v & v' \\ u & u' \end{vmatrix} = \frac{u_1}{v_1} = \frac{\phi_1(x)}{\psi_1(x)}.$$

$$\text{Likewise, } F''(x) = \frac{1}{v_1^2} \begin{vmatrix} v_1 & v'_1 \\ u_1 & u'_1 \end{vmatrix}; F'''(x) = \frac{1}{v_2^2} \begin{vmatrix} v_2 & v'_2 \\ u_2 & u'_2 \end{vmatrix}; \dots$$

$$\therefore F^{(n)}(x) = \frac{1}{v_{n-1}^2} \begin{vmatrix} v_{n-1} & v'_{n-1} \\ u_{n-1} & u'_{n-1} \end{vmatrix}, \text{ where } v_\lambda = v^{2^\lambda}, \quad v'_\lambda = 2^\lambda v^{2^\lambda - 1} v';$$